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MATHEMATICAL METHODS IN

WAVE PROPAGATION AND SCATTERING THEORY

C. H. Wilcox

FINAL TECHNICAL REPORT

June 1983

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## FINAL TECHNICAL REPORT

## Mathematical Methods in Wave Propagation and Scattering Theory

§1. Introduction. This report summarizes the research done for the Office of Naval Research by Professor Calvin H. Wilcox under contract

N00014-76-C-0276

University of Utah, September 1, 1975-June 30, 1983

The research was a direct continuation of research done for the Office of Naval Research by Professor Wilcox, Dr. John R. Schulenberger and Dr. William C. Lyford under the following contracts:

N00014-67-A-0209-0006

University of Arizona, January 1, 1969-December 31, 1969

N00014-67-A-0394-0002

University of Denver, September 1, 1969-August 31, 1971

N00014-67-A-0325-0006

University of Utah, September 1, 1971-August 31, 1975

Summaries of the research completed under these earlier contracts are contained in the Final Reports on these contracts.



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- §2. Background and Goals of the Research. Transient wave propagation phenomena are exploited in many branches of engineering and applied science. Examples include pulsed sonar and radar systems, pulsed laser beams and their applications; the use of seismic pulses in geophysical prospecting and ultrasonic pulses in medical imaging. The engineering literature provides an almost unlimited number of other examples. The continuing goal of the research performed under the contracts mentioned above was to develop mathematical methods which would provide both qualitative and quantitative understanding of transient wave propagation and scattering phenomena, with emphasis on acoustic and electromagnetic problems.
- §3. The Method of Asymptotic Wave Functions. Professor Wilcox has developed a method for analyzing transient wave propagation and scattering which he has called the method of asymptotic wave functions. The underlying physical idea is that transient waves in an unbounded structure will have an asymptotic or "final" form, for large values of the time, which is determined by the geometry of the structure and the nature of the medium filling it. This final form is described by a class of asymptotic wave functions which is independent of the precise form of the localized sources and scatterers that generate the waves. The wave sources and scatterers affect only the fine structure of the asymptotic wave functions. This suggests the following program
  - In each class of scattering problems select a suitable reference problem whose asymptotic wave functions can be calculated explicitly.
  - Determine the influence of localized scatterers by perturbation theory and an associated scattering operator which acts on the asymptotic wave functions.

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Asymptotic wave functions for dispersive media were introduced in 1973 in TSR #22, "Transient electromagnetic wave propagation along a dielectric-clad conducting plane" (published as "Transient electromagnetic wave propagation in a dielectric waveguide", Symposia Math. 18, 1976).

Asymptotic wave functions for non-dispersive media were introduced in 1974 in TSR #24, "Asymptotic wave functions and energy distributions for the d'Alembert equation" (published as Springer Lecture Notes in Mathematics V. 442, 1975). Since 1975 the method has been further developed and applied to a variety of propagation and scattering problems

§4. Projects Completed. The specific projects completed under the contract are described in detail in Technical Summary Reports 28 through 45, listed in §5 below. The different classes of physical problems treated in this work are outlined briefly in this section.

Asymptotic Wave Functions. The theory was extended to the most general anisotropic media in TSR 28. The specific case of electromagnetic waves in crystals was treated in TSR 33. An exposition of the method, with many examples, was presented in TSR 29. Applications to sonar and radar echoes were developed in TSRs 30, 35 and 41.

Asymptotic Energy Distributions. Asymptotic wave functions determine the ultimate distribution of energy in scattering problems. These were calculated for propagation in anisotropic media in TSR's 28, 29 and 33, for signals in stratified fluids in TSR's 36, 42 and 43, for signals scattered by periodic surfaces (diffraction gratings) in TSR 40 and for scattering in tubular waveguides in TSR's 29 and 45.

Propagation and Scattering in Tubular and Conical Waveguides. These were treated in TSR's 29, 31 and 45.

Propagation and Scattering in Periodic Structures. The one-electron theory of crystalline solids was developed in TSR 32. A complete theory of the scattering of steady-state and transient acoustic and electromagnetic waves by periodic surfaces was developed in TSR's 37, 39 and 40. This work will be published as a monograph entitled "Scattering Theory for Diffraction Gratings" by Springer-Verlag in the series Applied Mathematical Sciences (1983).

Propagation and Scattering in Stratified Fluids. A complete theory of the propagation and scattering of signals in stratified fluids whose sound speeds and densities are arbitrary functions of the depth was developed in TSR's 38, 42 and 43. This work will be published as a monograph entitled "Sound Propagation in Stratified Fluids" by Springer-Verlag in the series Applied Mathematical Sciences (1983).

Acoustic Imaging. Asymptotic wave functions were applied to a problem of medical imaging in TSR 44. An organ to be imaged is modelled as a fluid with variable sound speed and density. The scattering by the organ of ultrasonic waves produces measurable scattered fields whose asymptotic wave functions are functions of the unknown sound speed and density. A method, and algorithm, were developed for computing sound speed and density maps of the organ.

- §5. Technical Reports Completed Under ONR Contract No. N00014-76-C-0276.
  - TSR #28 <u>C. H. Wilcox</u>, "Asymptotic wave functions and energy distributions in strongly propagative media", March 1976.
  - TSR #29 <u>C. H. Wilcox</u>, "Spectral and asymptotic analysis of acoustic wave propagation", August 1976.

- TSR #30 C. H. Wilcox, "Sonar echo analysis" 1977.
- TSR #31 J.C. Guillot and C. H. Wilcox, "Steady-state wave propagation in simple and compound waveguides, 1977.
- TSR #32 C. H. Wilcox, "Theory of Bloch waves" 1977.
- TSR #33 <u>C. H. Wilcox</u>, "Electromagnetic signal propagation in crystals", 1977.
- TSR #34 Y. Saito, "Spectral representations for Schrödinger operators with long range potentials" December 1978.
- TSR #35 <u>C. H. Wilcox</u>, "The S-matrix and sonar echo analysis", 1979.
- TSR #36 <u>C. H. Wilcox</u>, "Transient acoustic wave propagation in an Epstein duct", November 1979.
- TSR #37 <u>C. H. Wilcox</u>, "Rayleigh-Bloch wave expansions for diffraction gratings I", March 1980.
- TSR #38 <u>C. H. Wilcox</u>, "Spectral analysis of sound propagation in stratified fluids", April 1980.
- TSR #39 <u>C. H. Wilcox</u>, "Rayleigh-Bloch wave expansions for diffraction gratings II", May 1980.
- TSR #40 <u>C. H. Wilcox</u>, "Scattering theory for diffraction gratings", August 1980.
- TSR #41 <u>C. H. Wilcox</u>, "Radar echo analysis by the singularity expansion method", January 1981.
- TSR #42 <u>C. H. Wilcox</u>, "Transient acoustic wave propagation in stratified fluids", July 1981.
- TSR #43 <u>C. H. Wilcox</u>, "Reflection, transmission and distortion of acoustic signals in stratified fluids", August 1981

- TSR #44 <u>C. H. Wilcox</u>, "Multiparameter acoustic imaging in the Born approximation", August 1982.
- TSR #45 <u>C. H. Wilcox</u>, "The S-Matrix and acoustic signal structure in simple and compound waveguides", December 1982.
- §6. Publications Completed Under ONR Contract No. N0014-76-C-0276.
  - C. H. Wilcox, Asymptocic wave functions and energy distributions in strongly propagative media, J. Math. pures et appl. 57, 275-321 (1978).
  - C. H. Wilcox, Spectral and asymptotic analysis of acoustic wave propagation, in Boundary Value Problems for Linear Evolution Partial Differential Equations, H. G. Garnir, Editor. D. Reidel Publ. Co., 385-473 (1977).
  - 3. <u>C. H. Wilcox</u>, Sonar echo analysis, Math. Methods in the Appl. Sci. 1, 70-88 (1979).
  - 4. J. C. Guillot and C. H. Wilcox, Steady-state wave propagation in simple and compound waveguides, Math. Zeit. 160, 89-102 (1978).
  - 5. <u>C. H. Wilcox</u>, Theory of Bloch waves, J. d'Anal. Math. <u>33</u>, 146-167 (1978).
  - 6. <u>C. H. Wilcox</u>, Electromagnetic signal propagation in crystals,
    Applicable Anal. <u>8</u>, 83-94 (1978).
  - 7. Y. Saito, Spectral representations for Schrödinger operators with long-range potentials, Springer Lecture Notes in Mathematics, #727, Springer-Verlag 1979.

- 8. .... Wilcox, The S-matrix and sonar echo structure,
  in Mathematical Methods and Applications of Scattering
  Theory, J. A. DeSanto, A. W. Sáenz and W. W. Zachary, Eds.,
  Lecture Notes in Physics, Springer-Verlag 1980.
- 9. <u>C. H. Wilcox</u>, Radar echo analysis by the singularity expansion method, Electromagnetics <u>1</u>, 481-491 (1981).
- 10. <u>C. H. Wilcox</u>, The S-matrix and acoustic signal structure in simple and compound waveguides, Applicable Anal.- to appear in 1983.
- 11. C. H. Wilcox, Multiparameter imaging in the Born approximation,

  Math. Methods in the Appl. Sci. to appear in 1983.
- 12. <u>C. H. Wilcox</u>, Scattering Theory for Diffraction Gratings, monograph, 162 pp. to appear in the series Applied Mathematical Sciences, Springer-Verlag, 1983.
- 13. <u>C. H. Wilcox</u>, Sound Propagation in Stratified Fluids, monograph, about 180 pp., to appear in the series Applied Mathematical Sciences, Springer-Verlag, 1983.

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